

MATHEMATICS

(Major)

Paper Code : MT201C

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer any six of the following questions :

$$2 \times 6 = 12$$

(a) Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{1 - \cos x}{x^2}, & \text{for } x \neq 0 \\ 1, & \text{for } x = 0 \end{cases}$$

at $x = 0$.

(b) Evaluate :

$$\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{\tan^2 x}$$

(c) State Maclaurin's theorem in Lagrange's
form of remainder.

(d) State Taylor's theorem for the function of two variables.

(e) Show that the pedal equation of the parabola $y^2 = 4(x+a)$ is $p^2 = ar$.

(f) Investigate the maximum and minimum values of the following polynomial :

$$2 - 9x + 6x^2 - x^3$$

(g) Show that if $f(x, y) = 2x^4 - 3x^2y + y^2$, then $f_{xx}f_{yy} - (f_{xy})^2 = 0$ at $(0, 0)$, but f has neither a maximum nor a minimum at $(0, 0)$.

(h) Find the asymptotes of the curve

$$x^2y^2 - a^2(x^2 + y^2) - a^3(x + y) + a^4 = 0$$

which are parallel to either axis.

Answer any four of the following questions : $12 \times 4 = 48$

2. (a) State and prove Lagrange's mean value theorem.

(b) Evaluate :

$$\lim_{x \rightarrow 0} \left(\tan \frac{\pi}{4} + x \right)^{1/x}$$

(c) If $y^{1/m} + y^{-1/m} = 2x$, then prove that

$$(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$$

$$5+3+4=12$$

3. (a) Consider the function f defined by

$$f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2}, & \text{where } x^2 + y^2 \neq 0 \\ 0, & \text{where } x^2 + y^2 = 0 \end{cases}$$

show that $f_{xy} \neq f_{yx}$ at $(0, 0)$.

(b) Evaluate :

$$\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$$

(c) If $y = e^{a \sin^{-1} x}$, then show that

$$(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + a^2)y_n = 0$$

$$4+3+5=12$$

4. (a) Show by the help of the Maclaurin's infinite expansion with Cauchy's form of remainder that

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots,$$

for $-1 < x \leq 1$

- (b) Show by applying mean value theorem that

$$\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}$$

if $0 < u < v$ and deduce that

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$$

- (c) If $u^3 + v^3 = x + y$ and $u^2 + v^2 = x^3 + y^3$, then show that

$$\frac{\partial(u, v)}{\partial(x, y)} = \frac{1}{2} \frac{(y^2 - x^2)}{2uv(u - v)}$$

4+5+3=12

5. (a) Find the maxima and minima of the function

$$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$$

- (b) Show that

$$\frac{\tan x}{x} > \frac{x}{\sin x}, \text{ for } 0 < x < \frac{\pi}{2}$$

- (c) Expand $e^{2x} \sin 3x$ in infinite series in power of x stating the condition under which the expansion is valid. 4+4+4=12

6. (a) Show that the maximum value of $x^2 \log\left(\frac{1}{x}\right)$ is $\frac{1}{2e}$.

- (b) If $u = \sin^{-1} \frac{x^2 + y^2}{x + y}$, then show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$$

- (c) Find the condition that the conics $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ shall cut orthogonally. 4+4+4=12

7. (a) State and prove Euler's theorem of homogeneous function of two variables.

- (b) Show that the tangent at (a, b) to the curve $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 = 2$ is $\frac{x}{a} + \frac{y}{b} = 2$.

- (c) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, then show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4 \sin^2 u) \sin 2u$$

4+3+5=12

$\sin u \cdot \cos^2 u$

(6)

8. (a) Find the asymptotes of

$$x^3 + x^2y - xy^2 - y^3 + x^2 - y^2 = 2$$

- (b) Examine the continuity of the following function at $(0, 0)$:

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & \text{when } x^2 + y^2 \neq (0, 0) \\ 0, & \text{when } x^2 + y^2 = (0, 0) \end{cases}$$

- (c) Find the envelop of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are variable parameters, connected by the relation $a^2 + b^2 = c^2$, c being a non-zero constant.

4+4+4=12

(7)

- (c) If ρ, ρ' be the radii of curvature at the ends of two conjugate diameters of an ellipse, then prove that

$$(\rho^{2/3} + \rho'^{2/3})(ab)^{2/3} = a^2 + b^2$$

4+4+4=12

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9. (a) Show that the radius of curvature at any point on the cardioid $r = a(1 - \cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$.

- (b) Find the asymptotes of

$$4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0$$

- (c) Find the condition that the plane $lx + my + nz = p$ may touch the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

4

9. (a) When is a cone said to be a right circular cone? Find the equation of the right circular cone whose vertex is at (α, β, γ) semi-vertical angle θ and the axis is given by

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

4

- (b) Find the equation of the cylinder whose generators are parallel to $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ and which passes through the curve $x=0$, $y^2 + z^2 = 4$.
4

- (c) Find the condition that two diameters of an ellipsoid should be conjugate.
4

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**UG Program (under NEP 2020)
2nd Semester Exam., 2025**

MATHEMATICS
(Major)

Paper Code : MT202C

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

1. Answer any six of the following questions :
 $2 \times 6 = 12$

- (a) Find the center and radius of the sphere

$$x^2 + y^2 + z^2 - 4x + 6y - 8z + 8 = 0$$

- (b) Show that the set $G = \{1, 0, -1\}$ does not form a group under usual addition.

- (c) Find the right circular cylinder whose radius is 1 and axis is z -axis.

- (d) Define order of a group and order of an element with example.

(2)

- (e) Find the equation of the plane passing through the point $(-4, 0, 4)$ and $(-1, -2, 3)$ and parallel to x -axis.
- (f) Determine the value of h for which the planes

$$3x - 2y - hz - 1 = 0$$

$$x + hy + 5z + 2 = 0$$

may be perpendicular to each other.

- (g) Show that the angle between the two planes

$$2x - y + 2z = 3$$

$$3x + 6y + 2z = 4$$

is $\cos^{-1}\left(\frac{4}{21}\right)$.

- (h) Show that identity element in a group $(G, *)$ is unique.

GROUP—B

Answer any four questions of the following :

$$12 \times 4 = 48$$

2. (a) Prove that the union of two subgroups is a subgroup if and only if one is contained in the other. 4
- (b) Prove that the centre of the group G is a subgroup of G . Find the centre of a group Q_B . 4

(3)

- (c) Prove that a multiplicative group G is Abelian if and only if

$$(ab)^{-1} = a^{-1}b^{-1}, \quad a, b \in G$$

4

3. (a) Show that the set of all real numbers R is a groupoid but not a semi-group under the operation $*$ defined by

$$a * b = a + 3b, \quad \forall a, b \in R$$

4

- (b) Show that the set $S = \{1, 2, 3, 4\}$ forms a group for operation multiplication modulo 5. 4

- (c) In the set Q of all rational numbers an operation $*$ is defined as follows :

$$a * b = a + b + ab \text{ for } a, b \in Q$$

Is $(Q, *)$ a group? Justify your answer. 4

4. (a) Prove that every subgroup of a cyclic group is cyclic. 4

- (b) Show that every field is an integral domain. Is the converse true? Justify. 4

- (c) Show that the order of an element of a group is the same as that of its element. 4

(4)

5. (a) Show that the set $\{1, -1, i, -i\}$ forms a cyclic group under multiplication. Find its generators. 4
- (b) Define commutative ring. If \mathbb{Z} be the set of all integers, prove that $(2\mathbb{Z}, +, \cdot)$ is a commutative ring with respect to addition and multiplication. 4
- (c) Prove that a finite integral domain is a field. 4
6. (a) Show that the straight lines whose direction cosines are given by the equation $2l+2m-n=0$ and $mn+nl+lm=0$ are perpendicular. 4
- (b) Find the equation of the sphere to which the planes $2x+3y-6z+14=0$ and $2x+3y-6z+42=0$ are tangents and its centre lies on the lines $2x+z=0, y=0$. 4
- (c) Find the equation of the image of the line
- $$\frac{x-1}{3} = \frac{y-3}{5} = \frac{z-4}{2}$$
- in the plane $2x-y+z+3=0$. 4

(5)

7. (a) Find the equation of the sphere whose centre is $(6, 3, -4)$ and which touches the axis of x . 4
- (b) Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y+z=0, z+x=0, x+y=0, x+y+z=c$ is $\frac{2c}{\sqrt{6}}$. 4
- (c) Find the equation of the sphere which passes through the points $(2, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 2)$ and has the least possible radius. 4
8. (a) Find the equation of the right circular cylinder whose axis is
- $$\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-1}{2}$$
- and radius 5. 4
- (b) Prove that the plane $ax+by+cz=0$ cuts the cone $yz+zx+xy=0$ in perpendicular lines if
- $$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$$
- 4

**UG Program (under NEP 2020)
2nd Semester Exam., 2025**

**MATHEMATICS
(Minor)**

Paper Code : MT201M

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

- 1.** Answer any six of the following questions :
 $2 \times 6 = 12$

- (a)* Find k if the equation

$$x^2 - y^2 + 2x + k = 0$$

represents a pair of straight lines.

- (b)* Prove that every orthogonal matrix has determinant ± 1 .

- (c)* Transform the equation $r^2 \sin 2\theta = 2a^2$ into Cartesian coordinates.

- (d)* Define a Hermitian matrix with an example.

(2)

- (e) Define isomorphism between two vector spaces with an example.
 (f) Find m if the straight line

$$\frac{x-1}{2} = \frac{y-5}{m} = \frac{z+2}{-1}$$

is parallel to the plane $x - 3y + 6z + 5 = 0$.

- (g) Find the equation of the sphere which has $(3, 4, -1)$ and $(1, 2, 3)$ as the end points of a diameter.
 (h) For the matrices

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

where $i^2 = -1$, prove that $AB = -BA$ and hence $(A+B)^2 = A^2 + B^2$.

GROUP—B

Answer any four of the following questions :

12×4=48

2. (a) Express the matrix

$$A = \begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}$$

as the sum of two matrices of which one is symmetrical and the other skew-symmetrical. 4

(3)

- (b) Show that the inverse of

$$\begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 2 & 2 & -1 \end{bmatrix} \text{ is } \frac{1}{4} \begin{bmatrix} 3 & -10 & -1 \\ -2 & 8 & 2 \\ 2 & -4 & -2 \end{bmatrix}$$

by elementary row operations. 4

- (c) Determine the values of α, β, γ when the matrix

$$\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$$

is orthogonal. 4

3. (a) Find the row reduced echelon form of

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & -4 & 5 \end{bmatrix}$$

and hence find the rank of it. 4

- (b) If

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

show that $A^3 - 5A^2 + 6A - 5I = 0$. Hence, find the value of A^{-1} . 4

(4)

- (c) Show that the commutative law of multiplication is not true for the matrices A and B , where

$$3A + B = \begin{bmatrix} 4 & 8 & 3 \\ 4 & 4 & -1 \\ -2 & 13 & 1 \end{bmatrix}$$

and

$$A - 2B = \begin{bmatrix} -1 & -4 & 6 \\ 1 & -1 & -2 \\ -3 & 2 & 2 \end{bmatrix} \quad 4$$

4. (a) Define a basis. Show that the vectors $(1, 0, 0)$, $(1, 1, 0)$ and $(1, 1, 1)$ form a basis for $R^3(R)$. $1+3=4$

- (b) Let $W = \{(x, 2y, 3z) : x, y, z \in R\}$, where R is the field of real numbers. Show that W is a subspace of V_3 over R . 4

- (c) If $T: R^3 \rightarrow R$ be defined such that $T(a, b, c) = 2a - 3b + 4c$, then show that T is a linear transformation. 4

5. (a) Use Cayley-Hamilton theorem to compute A^{-1} , where

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \quad 4$$

(5)

- (b) Given $T(x, y) = (2x + 3y, 4x + y)$, find the matrix of T and check if T is invertible. 4

- (c) When is a finite set of vectors said to be linearly dependent? Determine whether the vectors $\alpha_1 = (2, -1, 4)$, $\alpha_2 = (3, 6, 2)$, $\alpha_3 = (1, -1, 0)$ in R^3 form a linearly dependent or independent set. $1+3=4$

6. (a) The equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents two parallel straight lines. Show that the distance between them is

$$2 \sqrt{\frac{g^2 - ac}{a(a + b)}} \quad 4$$

- (b) Show that the angle through which the axes to be rotated so that the equation $x^2 + 2\sqrt{3}xy - y^2 = 2$ may be reduced to the form $x'^2 - y'^2 = 1$ is 30° . 4

- (c) Obtain the equation of the tangent to the conic $\frac{l}{r} = 1 - e \cos \theta$ at the point whose vectorial angle is α . 4

7. (a) Reduce the equation

$$x^2 + 2xy + 2y^2 + 6y + 9 = 0$$

to its canonical form and determine the type of the conic represented by it. 5

(6)

- (b) If PSP' and QSQ' be any two perpendicular focal chords of a conic, then show that

$$\frac{1}{SP \cdot SP'} + \frac{1}{SQ \cdot SQ'}$$

is a constant.

4

- (c) If $ax+by$ is transformed to $a'x'+b'y'$ due to rotation of axes, then show that

$$a^2 + b^2 = a'^2 + b'^2$$

3

8. (a) A variable plane is at a distance d from the origin O and cuts the coordinate axes at A, B, C . Show that the locus of the centroid of the tetrahedron $OABC$ is

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{d^2}$$

4

- (b) Find the distance of the point $(2, 3, -1)$ from the line

$$\frac{x-1}{2} = \frac{y+5}{1} = \frac{z+15}{-2}$$

4

- (c) Find the equation of the sphere for which the circle

$$x^2 + y^2 + z^2 + 2x - 4y + 2z + 5 = 0$$

$$x - 2y + 3z + 1 = 0$$

is a great circle.

4

(7)

9. (a) A plane passes through the point $(3, -3, 1)$. Also the line joining the points $(3, 4, -1)$ and $(2, -1, 5)$ is perpendicular to the plane. Find the equation of the plane.

4

- (b) Find the equation of the two tangent planes to the sphere

$$x^2 + y^2 + z^2 - 2x + 8y - 6z - 23 = 0$$

which are parallel to the plane
 $x + 5y + 3z - 3 = 0$.

5

- (c) Classify the surface given by

$$4x^2 + 9y^2 - 36z^2 = 36$$

3

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**UG Program (under NEP 2020)
4th Semester Exam., 2025**

**MATHEMATICS
(Major)**

Paper Code : MT401C

Full Marks : 60

Time : 3 hours

***The figures in the margin indicate full marks
for the questions***

GROUP—A

1. Answer any six of the following questions :

$2 \times 6 = 12$

- (a) State Cauchy-Riemann conditions.**
- (b) When is a function said to be an analytic function?**
- (c) Define linear span on a vector space V over the field F .**

(2)

- (d) Show that every subset of a linearly independent set of vectors is linearly independent.
- (e) Verify whether the set of vectors $(1, 2, 1), (2, 1, 0), (1, -1, 2)$ forms a basis of the vector space V_3 over the field of real numbers.
- (f) When is a set of vectors said to be orthogonal?
- (g) Verify Cayley-Hamilton theorem for the matrix

$$\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$$

- (h) Find the modulus and amplitude of the complex number

$$\frac{(1+i)(2+3i)}{(i-1)(2-3i)}$$

GROUP—B

Answer any four of the following questions :

$$12 \times 4 = 48$$

2. (a) If W_1 and W_2 be any two subspaces of a vector space V , then show that $W_1 \cap W_2$ is also a subspace of V in F .

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(Continued)

(3)

- (b) Show that the set of real functions f such that $f(x+1) = f(x)$ is a vector space over the field of real numbers.
- (c) Prove that a set of vectors S consisting of the n vectors $e_1 = (1, 0, 0, \dots, 0)$, $e_2 = (0, 1, 0, \dots, 0)$, \dots , $e_n = (0, 0, \dots, 0, 1)$ is a basis of $V_n(F)$.
- (d) Show that the set of vectors $S = \{1, 1+x, 1+x+x^2\}$ is a basis for V_3 over R . $3+3+4+2=12$

3. (a) Examine whether the following set of vectors is linearly dependent or independent :

$$\{(1, 0, 1), (1, 1, 0), (1, -1, 1)\}$$

- (b) Is the following set of vectors $\alpha = (a_1, a_2, \dots, a_n)$ in R^n subspace of R^n for all α such that a_2 is rational?

- (c) Let V be a vector space over the field F . Then show that the set S of non-zero vectors $\alpha_1, \alpha_2, \dots, \alpha_n \in V$ is linearly dependent if and only if some elements of S be a linear combination of the others. $4+3+5=12$

26M/409

(Turn Over)

(4)

4. (a) Prove that kernel of a linear transformation $T: V \rightarrow W$, is a subspace of V .
- (b) Let T be a linear transformation of R^2 into itself that maps $(1, 1)$ to $(-2, 3)$ and $(1, -1)$ to $(4, 5)$. Determine the matrix representing T w.r.t. the basis $\{(1, 0), (0, 1)\}$.
- (c) If the linear transformation T on $V_3(R)$ on the vectors of the standard ordered basis be $(0, 1, 3)$, $(2, -4, 0)$ and $(1, 0, 0)$ respectively, then find $T(a, b, c)$. $4+4+4=12$
5. (a) State and prove the rank nullity theorem of a linear transformation.
- (b) Examine whether the mapping $T: R^2 \rightarrow R^3$ defined by $T(x, y) = (x + 2y, 2x + y, x + y)$ is a linear transformation. Also find $\ker(T)$ and $\text{Im}(T)$.
- (c) Whether the transformation $T: R^2 \rightarrow R^2$ defined by $T(x, y) = (x + 1, y)$ is linear?
 $5+5+2=12$

(5)

6. (a) If α and β be any two vectors in an inner product space $V(F)$, then show that $|\langle \alpha, \beta \rangle| \leq \|\alpha\| \|\beta\|$, where $|\langle \alpha, \beta \rangle|$ denotes the modulus of the complex number $\langle \alpha, \beta \rangle$.
- (b) Find the eigenvalues and the corresponding eigenvectors of the matrix
$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$
- (c) State Cayley-Hamilton theorem. $5+6+1=12$
7. (a) Use the Gram-Schmidt process of orthonormalization to construct an orthonormal basis for the subspace of R^4 generated by $(1, 1, 0, 1)$, $(1, -2, 0, 0)$, $(1, 0, -1, 2)$.
- (b) Use Cayley-Hamilton theorem to compute A^{-1} , where
$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$
- (c) Show that the eigenvalues of a matrix and its transpose are the same.
 $5+4+3=12$

(6)

8. (a) If z be a complex number and $\frac{z+1}{z-i}$ be purely imaginary, then show that z lies on the circle whose centre is at the point $\frac{1}{2}(-1+i)$ and the radius is $\frac{1}{\sqrt{2}}$.

- (b) Express

$$\frac{-1+i\sqrt{3}}{1+i}$$

in polar form and then deduce the value of $\cos \frac{5\pi}{12}$.

- (c) If $\tan \log(x+iy) = a+ib$, where $a^2+b^2 \neq 1$, then prove that

$$\tan \log(x^2+y^2) = \frac{2a}{1-a^2-b^2}$$

5+4+3=12

9. (a) Deduce from the Gregory's series that

$$\pi = 2\sqrt{3} \left(1 - \frac{1}{3 \cdot 3} + \frac{1}{5 \cdot 3^2} - \frac{1}{7 \cdot 3^2} + \dots \right)$$

- (b) Show that the product of all the values of $(1+i\sqrt{3})^{3/4}$ is 8.

(7)

- (c) Show that the points representing the complex numbers z for which $|z+3|^2 - |z-3|^2 = 6$ lie on a straight line.

4+4+4=12

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**UG Program (under NEP 2020)
4th Semester Exam., 2025**

MATHEMATICS

(Major)

Paper Code : MT402C

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

1. Answer any six questions : 2×6=12

(a) If $I_n = \int_0^{\pi/2} \sin^n x dx$, then prove that

$$I_n = \frac{n-1}{n} I_{n-2}$$

**(b) State the fundamental theorem of
integral calculus.**

**(c) Find the unit vector normal to the
surface $x^2 - y^2 + z = 2$ at the point
(1, -1, 2).**

(2)

(d) If $f(a-x) = f(x)$, then show that

$$\int_0^a f(x) dx = 2 \int_0^{a/2} f(x) dx$$

(e) Prove that

$$\int_1^2 \vec{r} \times \frac{d^2 \vec{r}}{dt^2} dt = -14\hat{i} + 75\hat{j} - 15\hat{k}$$

where $\vec{r}(t) = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$.(f) Show that $(\vec{a} \cdot \vec{\nabla})\vec{r} = \vec{a}$.(g) If $\vec{a}(t)$ is a constant magnitude, then

$$\text{show that } \vec{a} \cdot \frac{d\vec{a}}{dt} = 0.$$

(h) If \vec{F} and \vec{G} are irrotational, then show that $\vec{F} \times \vec{G}$ is solenoidal.

GROUP—B

Answer any four questions :

12×4=48

2. (a) If $I_n = \int_0^1 (1-x^2)^n dx$, then prove that

$$(2n+1)I_n = 2nI_{n-1}$$

Hence find I_n .

6

(3)

(b) If $I_n = \int_0^{\pi/2} x^n \sin x dx$, n being a positive integer greater than 1, then show that

$$I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$$

Hence find the value of $\int_0^{\pi/2} x^5 \sin x dx$. 63. (a) Find the volume and the surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 + \cos \theta)$ about the base. 5(b) Show that the area bounded by $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. 3(c) Find the area of the surface generated by revolving about the Y-axis that part of the astroid $x = a\cos^3 \theta$, $y = a\sin^3 \theta$, that lies in the first quadrant. 44. (a) Show that $\iint \sqrt{4a^2 - x^2 - y^2} dx dy$ taken over the upper half of the circle $x^2 + y^2 - 2ax = 0$ is $\frac{4}{9}(3\pi - 4)a^3$. 4

(4)

- (b) Show that $\iiint_V xyz \, dx \, dy \, dz = \frac{1}{720}$, where V is the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$. 4
- (c) Show that $\iint x^2 y \, dx \, dy = \frac{1}{15} a^3 b^2$ taken over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 4

5. (a) Evaluate $\iiint_V dx \, dy \, dz$, where V is the ellipsoid $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$. 6

(b) Prove that

$$\iint_E \frac{\sqrt{a^2 b^2 - b^2 x^2 - a^2 y^2}}{\sqrt{a^2 b^2 + b^2 x^2 + a^2 y^2}} \, dx \, dy = \frac{\pi}{4} \left(\frac{\pi}{2} - 1 \right) ab$$

the field of integration being E , the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 6

6. (a) Prove that a necessary and sufficient condition that a proper vector $\vec{u}(t)$ always remains parallel to a fixed line is that $\vec{u} \times \frac{d\vec{u}}{dt} = \vec{0}$. 4

(5)

- (b) Find the directional derivative of the function $f(x, y, z) = xy\hat{i} + xy^2\hat{j} + z^2\hat{k}$ at the point $(2, 1, 2)$ in the direction of the outer normal to the sphere $x^2 + y^2 + z^2 = 9$. 3

- (c) Prove that $r^n \vec{r}$ is irrotational for all values of n but it is solenoidal only when $n = -3$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. 5

7. (a) Let $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the curve $x^2 + y^2 = 1, z = 0$. 3

- (b) If $\vec{\nabla}\phi = (x + 2y + 4z)\hat{i} + (2x - 3y - z)\hat{j} + (4x - y + 2z)\hat{k}$ then find the scalar function ϕ . 4

- (c) If $\vec{r} = 3t\hat{i} + 3t^2\hat{j} + 2t^3\hat{k}$, then calculate

$$\left[\frac{d\vec{r}}{dt} \frac{d^2\vec{r}}{dt^2} \frac{d^3\vec{r}}{dt^3} \right]$$

8. (a) Evaluate $\int_C xy \, dx + xy^2 \, dy$ by Stokes' theorem, where C is the square in xy -plane with vertices $(1, 0), (-1, 0), (0, 1)$ and $(0, -1)$. 4

(6)

- (b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where

$$\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$$

and the curve C is the rectangle in the xy -plane bounded by $y = 0$, $x = a$; $y = b$, $x = 0$.

4

- (c) If $\vec{F} = (3x^2y - z)\hat{i} + (xz^3 + y^4)\hat{j} - 2x^3z^2\hat{k}$,
then prove that

$$\text{grad div } \vec{F} = -6\hat{i} + 24\hat{j} - 32\hat{k}$$

at the point $(2, -1, 0)$.

4

9. (a) If $\vec{F} = x^2y\hat{i} + xz\hat{j} + 2yz\hat{k}$, then verify
that

$$\text{div curl } \vec{F} = 0$$

4

- (b) Verify Green's theorem in the plane for

$$\int_C [(x^2 - xy^2)dx + (y^2 - 2xy)dy]$$

where C is the square with vertices $(0, 0)$, $(2, 0)$, $(2, 2)$ and $(0, 2)$.

4

(7)

- (c) Evaluate $\int_S (y^2z^2\hat{i} + z^2x^2\hat{j} + x^2y^2\hat{k}) \cdot \hat{n} dS$,

where S is the part of the sphere $x^2 + y^2 + z^2 = 1$ above the xy -plane and bounded by this plane.

4

★ ★ ★

- (c) Four products are produced in three machines and their profit margin (machine to product) on sale are given in the following table :

		Products				Capacities
		P_1	P_2	P_3	P_4	
Machines	M_1	6	4	1	5	14
	M_2	8	9	2	7	18
M_3	4	3	6	2		7
Demands	6	10	15	8		

Demands of the products and capacities of the machines per week are shown in the table. Find a suitable production plan of the products in the machines to maximize the profit.

4

★ ★ ★

**UG Program (under NEP 2020)
4th Semester Exam., 2025**

MATHEMATICS
(Minor)

Paper Code : MT401M

Full Marks : 60

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

1. Answer any six of the following questions :

$2 \times 6 = 12$

- (a) Form a differential equation by eliminating the arbitrary constant from $y = ce^x$.

- (b) Solve :

$$x^2 \frac{dy}{dx} + y = 1$$

- (c) Examine whether the equation $(a^2 - 2xy - y^2)dx - (x + y)^2 dy = 0$ is exact or not.

(2)

- (d) Transform $x^2y'' + xy' + y = 0$ using $t = \log(x)$.

- (e) Reduce the following LPP in standard form :

Maximize $Z = 80x_1 + 55x_2$
subject to

$$4x_1 + 2x_2 \leq 40$$

$$2x_1 + 4x_2 \leq 32$$

and $x_1, x_2 \geq 0$

- (f) Eliminate the arbitrary constants A and B from the relation

$$y = Ae^x + Be^{-x} + x^2$$

- (g) Explain whether assignment problem is an LPP.

- (h) Find the extreme points from the following set :

$$S = \{(x, y) | x^2 + y^2 \leq 25\}$$

GROUP—B

Answer any four of the following questions :

12×4=48

2. (a) Solve, by the method of variation of parameters, the equation

$$\frac{d^2y}{dx^2} + 4y = 4 \tan 2x$$

4

(3)

- (b) Solve $(e^x + 1)y dy = (y^2 + 1)e^x dx$; given $y = 0$ when $x = 0$.

4

- (c) Write the general form of Clairaut's equation. Find the complete solution of

$$y = px + p - p^2 \quad 1+3=4$$

3. (a) Solve the equation

$$(D^2 - 2D + 1)y = xe^x$$

by the method of undetermined coefficients.

4

- (b) Find the orthogonal trajectories of the family of curves

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$

where a is a variable parameter.

4

- (c) Find the differential equation of all circles passing through the origin and having their centres on the x -axis.

4

4. (a) Solve

$$\frac{d^2y}{dx^2} - \frac{2}{x} \frac{dy}{dx} + \left(a^2 + \frac{2}{x^2} \right) y = 0$$

by reducing to normal form.

4

(4)

(b) Solve

$$x^6 \frac{d^2y}{dx^2} + 3x^5 \frac{dy}{dx} + a^2 y = \frac{1}{x^2}$$

by changing the independent variable. 4

(c) Solve

$$\frac{dy}{dx} + \frac{4x}{x^2 + 1} y = \frac{1}{(x^2 + 1)^3}$$

by the method of variation of parameters. 4

5. (a) Solve

$$x \frac{d^2y}{dx^2} + (x-1) \frac{dy}{dx} - y = x^2$$

by the method of operational factors. 4

(b) Solve

$$\begin{aligned} \frac{dx}{dt} - 7x + y &= 0 \\ \frac{dy}{dt} - 2x - 5y &= 0 \end{aligned}$$

4

(c) Find the eigenvalues and eigenfunctions for the differential equation

$$\frac{d^2y}{dx^2} + \lambda y = 0$$

which satisfies the boundary conditions

 $y(0) = 0$ and $y(\pi) = 0$. 4

(5)

6. (a) An electronic company manufactures two radio models each on a separate production line. The daily capacity of the first line is 60 radios and that of the second is 75 radios. Each unit of the first model uses 10 pieces of a certain electronic component, whereas each unit of second model requires 8 pieces of the same component. The maximum daily availability of the special components is 800 pieces. The profits per unit of models 1 and 2 are ₹ 500 and ₹ 400 respectively. Formulate the problem mathematically. 4

(b) Find a basic solution of the system of equations

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 6 \\ 2x_1 + x_2 + 4x_3 &= 4 \end{aligned}$$

Is the solution degenerate? 3+1=4

(c) Prove that the set of all feasible solutions of an LPP is a convex set. 4

7. (a) Solve the LPP by the Big-M method : 4

Minimize $Z = x_1 + x_2$

subject to

$$2x_1 + x_2 \geq 4$$

$$x_1 + 7x_2 \geq 7$$

$$x_1, x_2 \geq 0$$

(6)

- (b) Use duality to solve the LPP :

4

Minimize $Z = 3x_1 + x_2$

subject to

$2x_1 + 3x_2 \geq 2$

$x_1 + x_2 \geq 1$

$x_1, x_2 \geq 0$

- (c) Solve the following assignment problem :

4

	J_1	J_2	J_3	
P_1	10	20	30	
P_2	20	10	40	
P_3	50	30	20	

8. (a) Formulate the dual of the following LPP :

4

Maximize $Z = 2x_1 + 3x_2 + 4x_3$

subject to

$x_1 - 5x_2 + 3x_3 = 7$

$2x_1 - 5x_2 \leq 3$

$3x_2 - x_3 \geq 5$

 $x_1, x_2 \geq 0$ and x_3 is unrestricted in sign.

- (b) Prove that the transportation problem always has a feasible solution.

4

(7)

- (c) Find an optimal solution and corresponding cost of the following transportation problem :

4

	D_1	D_2	D_3	D_4	
O_1	6	1	9	3	70
O_2	11	5	2	8	55
O_3	10	12	4	7	90

85 35 50 45

9. (a) There are five machines and five jobs are to be assigned and the associated cost matrix is as follows. Find the proper assignment :

4

	Machines				
	I	II	III	IV	V
A	6	12	3	11	15
B	4	2	7	1	10
C	8	11	10	7	11
D	16	19	12	23	21
E	9	5	7	6	10

- (b) Prove that the dual of the dual of a primal LPP is the primal itself.

4

This booklet contains 15 printed pages. **Question Booklet No. :**

**Question Booklet for UG Program (under NEP 2020)
4th Semester Exam., 2025**

MATHEMATICS
(Inter-Disciplinary)

Full Marks : 60]

Paper Code : MT401ID

[Time : 3 Hours

Question Booklet SET No. : B

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

Read the following INSTRUCTIONS carefully :

1. Use black/blue dot pen only.
2. Fill in the particulars given below in this page.
3. Fill in the particulars (on the **Side 1**) of the OMR Answer Sheet as per Instructions contained in the OMR Answer Sheet.
4. The **SET No.** of this Question Booklet is **B**. Write the SET No. at the specific space provided in the OMR Answer Sheet.
5. There are **60 (sixty)** questions in this Question Booklet, each carrying **1 (one)** mark.
6. Each question or incomplete statement is followed by 4 (four) suggestive answers—[A], [B], [C] and [D] of which only **one** is correct. Mark the correct answer by darkening the appropriate circle.
7. Marking of **more than one** answer against any question will be treated as incorrect response and no mark shall be awarded.
8. **Any change in answer made or erased by using solid or liquid eraser will damage the OMR Answer Sheet resulting in rejection of the whole Answer Sheet by the computer. Therefore, do not change or erase once the answer is marked.**
9. No part of the Question Booklet shall be detached or defaced under any circumstances.
10. **Use of mobile phone, calculator, log table, compass, scale and any electronic gadget is strictly prohibited in the Examination Hall.**
11. **The OMR Answer Sheet must be returned to the Invigilator before leaving the Examination Hall.**
12. Adoption of unfair means in any form or violation of instruction as mentioned in Point No. **10** shall result in disciplinary action as per rules of the University.
13. The candidate must ensure that the Question Booklet and the OMR Answer Sheet are signed by the Invigilator.
14. **After opening the Question Booklet, check the total number of printed pages and report to the Invigilator in case of any discrepancy.**

Roll Number :

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OMR Answer
Sheet No. :

--

(As printed in the OMR Answer Sheet)

Verified and found correct

Full Signature of the Candidate

Signature of the Invigilator with date

1. Mean deviation about the mean for normal distribution is

[A] $\frac{2}{3}\sigma$

[B] $\frac{4}{5}\sigma$

[C] σ

[D] $\frac{3}{5}\sigma$

2. If $X \sim N(\mu, \sigma^2)$, then $Z^2 = \left(\frac{X-\mu}{\sigma}\right)^2$ is a

[A] beta distribution

[B] gamma distribution

[C] χ^2 variate

[D] None of the above

3. The matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 25 \end{bmatrix}$ is

[A] singular matrix

[B] non-singular matrix

[C] identity matrix

[D] null matrix

4. If $X \sim \beta(\mu, \nu)$, then the range of x is

[A] $0 \leq x < 2$

[B] $0 \leq x \leq 1$

[C] $0 \leq x < 1$

[D] $0 < x < 1$

5. The variance of the exponential distribution is

[A] $\frac{1}{\theta}$

[B] $\frac{1}{\theta^2}$

[C] θ

[D] e^θ

6. The rank of the matrix $\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$

is

[A] 1

[B] 2

[C] 3

[D] 4

7. If $X \sim \chi^2(n)$, then the relation between mean and variance is

[A] mean = variance

[B] mean = 2 variance

[C] 2 mean = variance

[D] None of the above

8. If X_1^2 and X_2^2 are independent χ^2 variates with n_1 and n_2 d.f. respectively, then $V = X_1^2 + X_2^2$ is also χ^2 variate with

[A] $n_1 n_2$ d.f.

[B] $n_1 - n_2$ d.f.

[C] $\frac{n_1}{n_2}$ d.f.

[D] $n_1 + n_2$ d.f.

9. If $\mu = \nu = 1$ and $X \sim \beta(\mu, \nu)$, then $f(x)$ is equal to

[A] 1

[B] 4

[C] 2

[D] 10

10. If $X \sim N(\mu, \sigma^2)$ and $Z = \frac{X - \mu}{\sigma}$ is a standard normal variate, then $\text{var}(Z)$ is equal to

[A] 0

[B] 1

[C] σ^2

[D] None of the above

11. Who developed Python programming language?

[A] Wick van Rossum

[B] Rasmus Lerdorf

[C] Guido van Rossum

[D] Niene Stom

12. All keywords in Python are in

[A] capitalized

[B] lower case

[C] upper case

[D] None of the above

13. What will be the value of the following Python expression?

```
print (4 + 3 % 5)
```

[A] 7

[B] 2

[C] 4

[D] 1

14. Which keyword is used for function in Python language?

[A] Function

[B] def

[C] Fun

[D] Define

15. What will be the output of the following Python expression if $x = 56.236$?

```
print ("%.2f" % x)
```

[A] 56.236

[B] 56.23

[C] 56.0000

[D] 56.24

16. Which of the following functions is a built-in function in Python?

[A] factorial()

[B] print()

[C] seed()

[D] sqrt()

17. What are the values of the following Python expressions?

```
print (2 **(3 ** 2))
```

```
print ((2 ** 3) ** 2)
```

```
print (2 ** 3 ** 2)
```

[A] 512, 64, 512

[B] 512, 512, 6512

[C] 64, 512, 64

[D] 64, 64, 64

18. What arithmetic operators **cannot** be used with strings in Python?

[A] *

[B] -

[C] +

[D] All of the above

19. What is the output of the following?

```
print (math.pow(3, 2))
```

[A] 9.0

[B] 6

[C] 8.0

[D] None of the above

20. What will be the output of the following Python code snippet?

```
z = set ('abc$de')
print ('a' in z)
```

[A] Error

[B] True

[C] False

[D] No output

21. Which of the following is used to create an empty set in Python?

[A] ()

[B] []

[C] {}

[D] set()

22. Which one of the following is *not* a keyword in Python language?

[A] pass

[B] eval

[C] assert

[D] nonlocal

23. What will be the output of the following Python function?

```
print (len (["hello", 2, 4, 6]))
```

[A] Error

[B] 6

[C] 4

[D] 3

24. What will be the output of the following Python function?

```
print (min (max (False, -3, -4), 2, 7))
```

[A] -4

[B] -3

[C] 2

[D] False

25. Which of the following is the truncation division operator in Python?

[A] |

[B] //

[C] /

[D] %

26. Python supports the creation of anonymous functions at runtime, using a construct called

[A] pi

[B] anonymous

[C] lambda

[D] None of the above

27. Which of the following characters is used to give single-line comments in Python?

[A] //

[B] #

[C] !

[D] /*

28. Which of the following is used to define a block of code in Python language?

[A] Indentation

[B] Key

[C] Bracket

[D] All of the above

29. Which of the following is the correct extension of the Python file?

[A] .python

[B] .pl

[C] .py

[D] .p

30. Is Python case sensitive when dealing with identifiers?

[A] No

[B] Yes

[C] Machine dependent

[D] None of the above

31. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A^T = I$, then the value of α is

[A] $\frac{\pi}{6}$

[B] $\frac{\pi}{3}$

[C] π

[D] $\frac{\pi}{2}$

32. If the rank of the matrix $A = \begin{bmatrix} 2 & 1 & 4 \\ 1 & \alpha & 2 \\ 4 & 0 & \alpha+2 \end{bmatrix}$ is 2, then α is equal to

[A] $6, \frac{1}{2}$

[B] $4, \frac{1}{2}$

[C] $5, \frac{1}{2}$

[D] $0, \frac{1}{2}$

33. The matrix $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a

[A] square matrix

[B] diagonal matrix

[C] unit matrix

[D] None of the above

34. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is

[A] 3

[B] 0

[C] -1

[D] 1

35. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$ is

[A] 3

[B] 2

[C] 1

[D] 0

36. What is the order of the matrix B , if $[3 \ 4 \ 2]B = [2 \ 1 \ 0 \ 3 \ 6]$?

[A] 3×2

[B] 1×5

[C] 3×5

[D] 5×3

37. The value of the determinant of an orthogonal matrix is

[A] 2

[B] ± 1

[C] -1

[D] 0

38. If $A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & K \end{bmatrix}$ is an orthogonal matrix, then the value of K is

[A] ± 2

[B] ± 3

[C] ± 1

[D] 0

39. A complex $n \times n$ matrix A , satisfying the relation $AA^* = I$, is called

[A] Hermitian

[B] skew-Hermitian

[C] unitary

[D] orthogonal

40. If the matrix $A = \begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$

is singular, then x is equal to

[A] 1, 3

[B] 0, 2

[C] 1, 0

[D] 0, 3

41. The rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

is

[A] 2

[B] 1

[C] 3

[D] 0

42. If $A + I = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$, then $(A + I)(A - I)$ is

[A] 0

[B] $\begin{bmatrix} -4 & 3 \\ -1 & -3 \end{bmatrix}$

[C] $\begin{bmatrix} 3 & -4 \\ -3 & -1 \end{bmatrix}$

[D] None of the above

43. The inverse of a matrix, if exists, is

[A] unique

[B] not unique

[C] identity matrix

[D] None of the above

44. The matrix $A = \frac{1}{3} \begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is

[A] orthogonal

[B] symmetric

[C] skew-symmetric

[D] None of the above

45. If the system of equations $2x + y - z = 12$,

$x - y - 2z = -3$ and $3y + 3z = K$ is

consistent, then

[A] $K \neq 18$

[B] $K \neq -18$

[C] $K = -18$

[D] $K = 18$

46. If A is a Hermitian matrix, then iA is

[A] Hermitian

[B] skew-Hermitian

[C] orthogonal

[D] unitary

47. The matrix $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ is

[A] idempotent

[B] involuntary

[C] nilpotent

[D] None of the above

48. $[AB]^{-1}$ is equal to

[A] AB

[B] $A^{-1}B^{-1}$

[C] $B^{-1}A^{-1}$

[D] None of the above

49. If $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$, then find the value of A .

[A] $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

[B] $\begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

[C] $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

[D] $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

50. AA^{-1} is equal to the matrix

[A] A

[B] A^{-1}

[C] I

[D] 0

51. The m.g.f. of normal distribution is

[A] $e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

[B] $e^{\frac{1}{2}t^2}$

[C] e^{2t^2}

[D] $e^{t/2}$

52. For standard normal variate, $M_X(t)$ is equal to

[A] e

[B] $2e$

[C] $e^{\frac{1}{2}t^2}$

[D] e^{t^2}

53. For normal distribution, the relation among mean, median and mode is

[A] mean > median > mode

[B] mean < median < mode

[C] mean = median \neq mode

[D] mean = median = mode

54. If $f(x)$ be the p.d.f. for normal distribution, then the range of x is

[A] $-\infty < x < \infty$

[B] $-1 < x < 0$

[C] $0 < x < 1$

[D] None of the above

55. The mean of exponential distribution is

[A] $\frac{1}{\theta}$

[B] $\frac{1}{\theta^2}$

[C] θ

[D] e^θ

56. If $X \sim N(\mu, \sigma^2)$, then p.d.f. of $\frac{1}{2} \left(\frac{X - \mu}{\sigma} \right)^2$ is

[A] beta distribution

[B] $\gamma\left(\frac{1}{2}\right)$

[C] χ^2 distribution

[D] None of the above

57. If $X \sim N(\mu, \sigma^2)$, then the mean and variance of X is

[A] (μ, σ^2)

[B] (μ, σ)

[C] (μ^2, σ)

[D] (μ^2, σ^2)

58. If $X \sim N(\mu, \sigma^2)$, then the value of β_2 is

[A] 1

[B] 3

[C] 0

[D] 2

59. The m.g.f. of χ^2 distribution is

[A] $(1 - 2t)^{-n/2}$

[B] $(1 + 2t)^{-n/2}$

[C] $(1 + t)^{n/2}$

[D] None of the above

60. The mean of χ^2 distribution is

[A] 1

[B] n

[C] $2n$

[D] None of the above

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK